3 ELECTRON BALLISTICS

CHAPTER

LEARNING OBJECTIVES

After studying this chapter, the student should be able to:

- Examine the behaviour of electrons under the influence of electric and magnetic fields.
- Describe and analyze the relation between the motion of electrons under the influence of electric and magnetic fields, and the operations of electronic devices.

3.1 INTRODUCTION

The operation of an electronic device depends upon the motion of electrons under the influence of electric and magnetic fields. The behaviour of an electron under the influence of these fields is termed electron ballistics. This chapter deals with the study of basic properties of matter. This chapter starts with simple definitions, motion of particles in simple paths under uniform fields to complex paths under varying fields.

3.2 CHARGED PARTICLES

FLASH CARD

What You Learn Here
Discuss the nature of charge particles.

Electron forms the most fundamental charged particle. An electron is negatively charged; its magnitude is 1.602×10^{-19} coulomb. The number of electrons per coulomb is 6×10^{18} and 1 A of current represents the motion of 6×10^{18} electrons per second. The mass of an electron is 9.107×10^{-31} kg. A direct measurement of mass of an electron is not possible but the ratio of electron's charge to mass can be determined by a number of experiments and found to be 1.759×10^{11} C/kg. The mass of an electron can be deduced from this value.

The positive ion possesses a charge that is an integral multiple of the charge of an electron and is positive. If a positive ion is singly ionized, it has a charge equal to that of an electron and if it is doubly ionized, its charge is twice that of an electron. The mass of a hypothetical atom of atomic weight unity is, as per definition, taken as one-sixteenth of the mass of monoatomic oxygen and has been calculated to be 1.660×10^{-27} kg. Hence, the mass of any atom, in kilogram, can be calculated by multiplying the atomic weight of the atom by 1.660×10^{-27} kg.

The radius of an electron is very small. It is estimated to be much closer to 3.8×10^{-10} m. The radius of an atom is estimated as 10^{-10} m. These dimensions being very small, all charges are considered to be mass points.

3.3 FORCE, FIELD INTENSITY, POTENTIAL, AND ENERGY

FLASH CARD

What You Learn Here
Define force, field intensity, potential, and energy.

- ***Force*** Charles Coulomb stated that the force between two very small objects separated by a distance which is large compared to their size is proportional to the charge on each and inversely proportional to the square of the distance between them, or

$$F = k\frac{Q_1 Q_2}{d^2} \tag{3.1}$$

where Q_1 and Q_2 are the charges, d the separation, and k the proportionality constant. In SI units, Q is measured in coulomb (C), d in metre, and the force should be in Newton (N). This will be achieved if the constant of proportionality k is written as

$$k = \frac{1}{4\pi\varepsilon_o} \tag{3.2}$$

where ε_o is the permittivity of free space and has a value of 8.854×10^{-12} F/m.

Coulomb's law shows that the force between two charges of 1 C each, separated by one metre, is 8.9×10^9 N and that between two electrons is 230×10^{-13} N.

- ***Field Intensity*** The electric field intensity is defined as the force on a unit positive charge, or conversely, the force on a unit positive charge at any point in an electric field is the electric field intensity E at that point. The unit of electric field intensity is *volt per metre* (V/m).

If a charge is moved against the electric field, a force equal to and opposite to the force exerted by the field is needed, and this requires expending energy or doing work. On the other hand, if the charge is moved in the direction of the field, the energy expenditure turns out to be negative; we do not do the work, the field does. When a charge q is moved in an electric field E, the force on q due to the electric field is

$$F_q = qE \tag{3.3}$$

The path of a charged particle in an electric field can be calculated by relating the force, given by Eq. (3.3), to the mass and the acceleration of the particle by Newton's second law of motion. Thus,

$$F_q = qE = ma = m\frac{dv}{dt} \tag{3.4}$$

PROBE

How can the path of a charged particle be calculated?

where m = mass, kg; a = acceleration, m/s^2; and v = velocity, m/s.

The solution of this equation with appropriate initial conditions gives the path of the particle resulting from the action of the electric forces.

- ***Potential*** Consider a parallel-plate capacitor as shown in Fig. 3.1. A difference of potential is applied between the two plates as shown. The separation between the two plates is much smaller in comparison with the dimensions of the plates. This ensures uniform electric field between the plates, the lines of force pointing along negative Z-direction. At time $t = 0$, the initial velocity v_z is equal to zero. Since there is no force along the X- or Y-directions, the acceleration along these directions is zero and since the initial velocity is zero, the particle will not move in these axes.

By definition, the potential V (in volt) of point z with respect to the point z_o is the work done against the field in taking a unit positive charge from z_o to z.

Thus, $$V = -\int_{z_o}^{z} E \cdot dz \tag{3.5}$$

The integral in Eq. (3.5) can be evaluated to the form

$$-\int_{z_o}^{z} E_z \cdot \mathrm{d}z = -E_z(z - z_o) = V \tag{3.6}$$

$$E_z = -\frac{V}{z - z_o} = -\frac{V}{d} \tag{3.7}$$

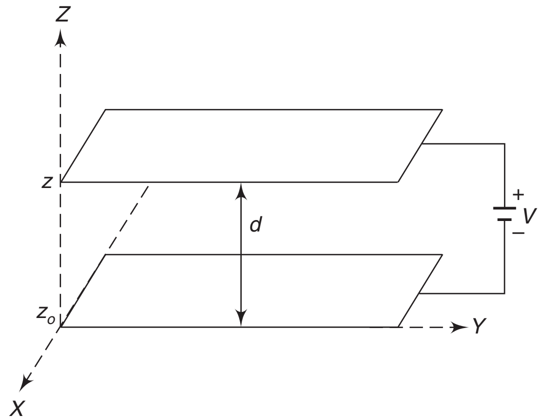

Fig. 3.1 *Potential difference across two parallel plates*

Thus, from Eq. (3.7), the electric field intensity is negative of the ratio of applied potential difference between the two plates to the separation between the two plates. Equation (3.7) is valid for uniform electric fields; however, for fields varying with distance between the plates the field is given by

$$E_z = -\frac{dV}{dz} \tag{3.8}$$

The negative sign in Eq. (3.8) signifies that the orientation of the electric field is from higher potential point to lower potential point.

- ***Potential Energy*** For an electron between two parallel plates, the Newton's second law can be applied. Then,

$$-q\,E_z = ma \tag{3.9}$$

$$-\frac{q}{m}E_z = \frac{dv_z}{dt} \tag{3.10}$$

The velocity of the particle in the Z direction is defined as the rate of change of displacement of the particle in z direction, i.e.,

$$v_z = \frac{dz}{dt}$$

$$dz = v_z\,dt \tag{3.11}$$

Multiplying Eq. (3.10) by Eq. (3.11) and integrating,

$$-\frac{q}{m}\int_{z_o}^{z} E_z\,dz = \int_{v_{oz}}^{v_z} \frac{dv_z}{dt} \cdot v_z \cdot dt$$

$$-\frac{q}{m}\int_{z_o}^{z} E_z\,dz = \int_{v_{oz}}^{v_z} v_z \cdot dv_z \tag{3.12}$$

By virtue of Eq. (3.5), Eq. (3.12) integrates to

$$\frac{q}{m}V = \frac{1}{2}(v_z^2 - v_{oz}^2)$$

$$qV = \frac{1}{2}m(v_z^2 - v_{oz}^2) \tag{3.13}$$

where the energy qV is expressed in joule.

Considering any two points A and B in space, with B at a higher potential than A by V_{BA}, Eq. (3.13) can be stated in a more general form,

$$q\,V_{BA} = \frac{1}{2}m\,V_A^2 - \frac{1}{2}m\,V_B^2 \tag{3.14}$$

where q is the charge in *coulomb*, qV_{BA} in *joule*, and V_A and V_B are the initial and final velocities in *metre per second.*

By definition, the potential energy between two points equals the potential multiplied by the charge. Equation (3.14) is a statement of *Law of Conservation of Energy*. The left-hand side of Eq. (3.14) is the rise in potential energy from A to B and the right-hand side is drop in kinetic energy from A to B. Thus, the rise in potential energy equals the drop in kinetic energy. Equation (3.14) is not valid for time-varying fields.

EXAMPLE 3.1

Find the speed and the kinetic energy of an electron after it has moved through a potential difference of 5000 V.

Solution The speed of the electron, $v = \sqrt{\dfrac{2qV}{m}} = \sqrt{\dfrac{2\times 1.602\times 10^{-19}\times 5000}{9.1\times 10^{-31}}} = 4.2\times 10^{7}$ m/s

The kinetic energy = $q \times V = 1.602 \times 10^{-19} \times 5000 = 8 \times 10^{-16}$ J = 5000 eV

EXAMPLE 3.2

A charged particle having mass equal to 1000 times of an electron and a charge same as that of an electron is accelerated through a potential difference of 1000 V. Calculate the velocity attained by the charged particle and kinetic energy in terms of electron volt and joule.

Solution The mass of the charged particle = 1000 times the mass of an electron

$= 1000 \times 9.1 \times 10^{-31} = 9.1 \times 10^{-28}$ kg

The charge of the particle = 1.602×10^{-19} C

Therefore, the velocity, $v = \sqrt{\dfrac{2qV}{m}} = \sqrt{\dfrac{2\times 1.602\times 10^{-19}\times 1000}{9.1\times 10^{-28}}} = 0.596\times 10^{6}\ \text{ms}^{-1}$

Kinetic energy = $q \times V$

$= 1.602 \times 10^{-19} \times 1000 = 1.6 \times 10^{-16}$ J = 1000 eV

3.4 TWO-DIMENSIONAL MOTION OF ELECTRON

The following assumptions are made in the following analysis:

1. The charge density is low enough so that the mutual repulsive force may be neglected.
2. The movement of the charged particle is in high vacuum so that no collision with gas atoms or ions occur.
3. The mass of the charged particle is extremely small so that the gravitational force may be neglected as compared to the forces exerted by the fields.

FLASH CARD

What You Learn Here
Examine the motion of an electron in an electric field.

Consider a parallel-plate capacitor with uniform electric field in between the plates. If an electron enters the region between the two plates with an initial velocity in the +*X*-direction, the electron is made to move in a parabolic path. Referring to Fig. 3.2, the electric field *E* is in the –*Y*-direction, and no other fields are existing in other directions.

The initial conditions, at $t = 0$, are

$$\begin{aligned} v_x &= v_{ox}, & x &= 0 \\ v_y &= 0, & y &= 0 \\ v_z &= 0, & z &= 0 \end{aligned}$$

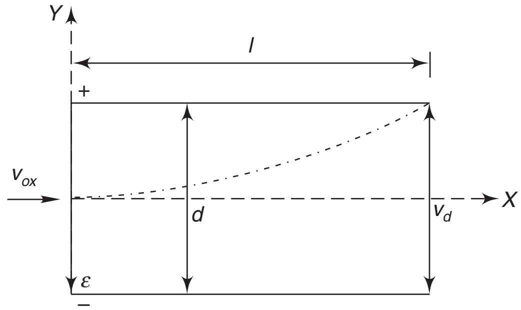

Fig. 3.2 *Motion of an electron in an electric field*

Let us analyse the fields in the different axes.

- ***Z-axis*** Since no field is present in this direction, the force is also zero (since $F = \varepsilon E$). As the force is zero, the acceleration along the *Z*-direction is also zero (since $F = ma$). The zero acceleration means no change in the velocity component in *Z*-direction. Since the initial velocity $v_z = 0$, the final velocity must also be zero. Hence, there is no movement of electrons in the *Z*-direction.

- ***X-axis*** The electron moves with a constant velocity of v_{ox} in the *X*-direction, for the same reasons given above.

- ***Y-axis*** The field is in the –*Y*-direction, which is uniform. Hence, *F* is also uniform and constant, which makes the electrons have a constant acceleration along the *Y*-direction. The motion in the *Y*-direction is given by

$$v_y = a_y t \qquad y = \frac{1}{2} a_y t^2 \tag{3.15}$$

where a_y is the acceleration along the *Y*-direction, and using Newton's second law,

$$a_y = -q\frac{E_y}{m} = \frac{qV_d}{md} \tag{3.16}$$

where V_d is the potential across the plates and *d* is the separation. The above equations indicate an upward motion of the electron in the region between the plates. The velocity component v_y changes from point to point, whereas the velocity component v_x remains unchanged. The distance covered by an electron along the *X*-direction is given by

$$x = v_{ox}\, t \tag{3.17}$$

Combining Eqs (3.15) and (3.17),

$$y = \frac{1}{2}a_y t^2 = \frac{1}{2}a_y\left(\frac{x}{v_{ox}}\right)^2$$

$$y = \frac{1}{2}\left(\frac{a_y}{v_{ox}^2}\right)x^2$$

$$y = kx^2 \quad \text{where} \quad k = \frac{1}{2}\left(\frac{a_y}{v_{ox}^2}\right) \tag{3.18}$$

Equation (3.18) shows that the electron moves in a parabolic path in the region between the plates.

EXAMPLE 3.3

In a parallel-plate diode, anode is made 350 V positive with respect to the cathode and is 6 mm from it. An electron is emitted from the cathode with an initial velocity of 3×10^6 m/s in the direction of the anode. Calculate the velocity and the time of travel of the electron (a) when it is midway between cathode and anode, and (b) on reaching the anode.

Solution Given $V = 350$ V, $d = 6$ mm, $v_{ox} = 3 \times 10^6$ m/s

Therefore, $E = V/d = 350/(6 \times 10^{-3}) = 58.33 \times 10^3$ V/m

$$a_x = qE/m = 1.026 \times 10^{16} \text{ m/s}^2$$

We know that,

$$x = v_{ox}t + 0.5\, at^2$$

$$v_x = v_{ox} + a_x t$$

(a) Consider $x = 3 \times 10^{-3}$ m

$$3 \times 10^{-3} = 3 \times 10^6 t + 5.13 \times 10^{15}\, t^2$$

$$t^2 + 5.85 \times 10^{-10}\, t - 5.85 \times 10^{-19} = 0$$

Solving this equation,

$$t = 5.26 \times 10^{-10} \text{ s}$$

Therefore, $v_x = v_{ox} + a_x t = 3 \times 10^6 + 1.026 \times 10^{16}\,(5.264 \times 10^{-10})$

$$= 8.4 \times 10^6 \text{ m/s}$$

(b) Consider $x = 6 \times 10^{-6}$ m

$$t^2 + 5.85 \times 10^{-10}\, t - 1.17 \times 10^{-18} = 0$$

Solving this equation, $t = 8.28 \times 10^{-10}$ s

Therefore, $v_x = 3 \times 10^6 + 8.28 \times 10^{-10}\,(1.026 \times 10^{16}) = 11.5 \times 10^6$ m/s

EXAMPLE 3.4

In a parallel diode, the cathode and anode are spaced 5 mm apart and the anode is kept at 200 V d.c. with respect to cathode. Calculate the velocity and the distance travelled by an electron after a time of 0.5 ns, when (a) the initial velocity of an electron is zero, and (b) the initial velocity is 2×10^6 m/s in the direction towards the anode.

Solution Given $V = 200$ V, $d = 5$ mm, Time of travel by an electron, $t = 0.5$ ns.

Therefore, $$E = \frac{V}{d} = \frac{200}{5\times10^{-3}} = 40\times10^{3}\ \text{V/m}$$

$$a_x = \frac{qE}{m} = \frac{1.602\times10^{-19}\times40\times10^{3}}{9.1\times10^{-31}} = 70.33\times10^{14}\ \text{m/s}^2$$

We know that

Velocity of an electron, $v_x = v_{ox} + a_x t$

Distance travelled by an electron, $x = v_{ox} + 0.5at^2$

(a) When the initial velocity of an electron is zero

$$v_x = v_{ox} + a_x t = 0 + 70.33\times10^{14}\times0.5\times10^{-9} = 3.5165\times10^{6}\ \text{m/s}$$

$$x = v_{ox}t + 0.5\,at^2 = 0 + 0.5\times70.33\times10^{14}\times(0.5\times10^{-9})^2 = 0.879\times10^{-3}\ \text{m}$$

(b) When the initial velocity of an electron is $v_{ox} = 2\times10^{6}$ m/s

$$v_x = v_{ox} + a_x t = 2\times10^{6} + 70.33\times10^{14}\times0.5\times10^{-9} = 5.51\times10^{6}\ \text{m/s}$$

$$x = v_{ox}t + 0.5\,at^2 = 2\times10^{6}\times0.5\times10^{-9} + 0.5\times70.33\times10^{14}\times(0.5\times10^{-9})^2$$
$$= 1.879\times10^{-3}\ \text{m}$$

EXAMPLE 3.5

Two plane parallel plates A and B are placed 3 mm apart and potential of B is made 200 V positive with respect to plate A. An electron starts from rest from the plate A. Calculate (a) the velocity of the electron on reaching the plate B, (b) time taken by the electron to travel from the plate A to the plate B, and (c) kinetic energy of the electron on reaching the plate B.

Solution (a) The electron starts from rest at the plate A, therefore, the initial velocity is zero. The velocity of the electron on reaching the plate B is

$$v = \sqrt{\frac{2qV}{m}} = \sqrt{\frac{2(1.602\times10^{-19})200}{(9.1\times10^{-31})}} = 8.38\times10^{6}\ \text{m/s}$$

(b) Time taken by the electron to travel from the plate A to the plate B can be calculated from the average velocity of the electron in transit. The average velocity is

$$v_{\text{average}} = \frac{\text{Initial velocity} + \text{Final velocity}}{2} = \frac{0 + 8.38\times10^{6}}{2} = 4.19\times10^{6}\ \text{m/s}$$

Therefore, the time taken for travel is

$$\text{Time} = \frac{\text{Separation between the plates}}{\text{Average velocity}} = \frac{3\times10^{-3}}{4.19\times10^{6}} = 0.71\times10^{-9}\ \text{s}$$

(c) Kinetic energy of the electron on reaching the plate B is

$$\text{Kinetic energy} = q\,V = (1.602\times10^{-19})\,200 = 3.2\times10^{-17}\ \text{J}$$

EXAMPLE 3.6

Two plane parallel plates A and B are placed 8 mm apart and the plate B is 300 V more positive than the plate A. The electron travels from the plate A to the plate B with an initial velocity of 1×10^6 m/s. Calculate the time of travel.

Solution The speed acquired by the electron due to the applied voltage is

$$v = \sqrt{v_{\text{initial}}^2 + \frac{2qV}{m}} = \sqrt{(1 \times 10^6)^2 + \frac{2(1.602 \times 10^{-19})300}{9.1 \times 10^{-31}}} = 10.33 \times 10^6 \text{ m/s}$$

The average velocity,

$$v_{\text{average}} = \frac{v_{\text{initial}} + v_{\text{final}}}{2} = \frac{1 \times 10^6 + 10.33 \times 10^6}{2} = 5.665 \times 10^6 \text{ m/s}$$

Therefore, the time for travel $= \dfrac{\text{separation between plates}}{v_{\text{average}}} = \dfrac{8 \times 10^{-3}}{5.665 \times 10^6} = 1.4 \times 10^{-9}$ s

EXAMPLE 3.7

An infinitely large parallel plane plates are spaced 0.8 cm apart. The voltage at one of the plates is raised from 0 to 5 V in 1 ns at a uniform rate, with respect to the other. After duration, the potential difference between the plates is suddenly dropped to 0 V and remains the same thereafter. Find (a) the position of the electron, which started with zero initial velocity from the negative plate, when the potential difference drops to zero volt, and (b) the total time of transit of the electron from the cathode to the anode.

Solution The electric field intensity,

$$E = -\frac{5t}{10^{-9} d} = -\frac{5t}{10^{-9} \times 1 \times 10^{-2}} = 5 \times 10^{11}\, t \,(\text{for } 0 < t < t_1)$$

$$= 0 \quad (\text{for } t_1 < t < \infty)$$

The velocity of electron, $v_x = -\int \frac{qE_x}{m} dt + C \quad$ (C is the initial velocity)

$$= \int 5 \times 10^{11} \frac{q}{m} t \, \mathrm{d}t = 5 \times 10^{11} \frac{q}{m} \cdot \frac{t^2}{2}$$

The distance travelled by the electron,

$$d = \int v_x \, \mathrm{d}t = \int 5 \times 10^{11} \frac{q}{m} \cdot \frac{t^2}{2} \mathrm{d}t + C \qquad (C = 0)$$

$$= 5 \times 10^{11} \frac{q}{m} \cdot \frac{t^2}{2}$$

(a) The position of the electron after 1 ns,

$$d = (5 \times 10^{11}) \cdot (1.76 \times 10^{11}) \cdot \frac{(1 \times 10^{-9})^3}{6}$$

$$= 14.7 \times 10^{-6} \text{ m} = 14.7\ \mu\text{m}$$

(b) The rest of the distance to be covered by the electron = 0.8 cm – 14.7 µm = 0.799 cm

Since the potential difference drops to zero volt, after 1 ns, the electron will travel the distance of 0.799 cm with a constant velocity of

$$v_x = 5\times10^{11}\,\frac{q}{m}\cdot\frac{t^2}{2} = (5\times10^{11})\cdot(1.76\times10^{11})\cdot\frac{(1\times10^{-9})^3}{6} = 44\times10^3 \text{ m/s}$$

Therefore, the time $$t_2 = \frac{d}{v_x} = \frac{0.799\times10^{-2}}{44\times10^3} = 1.816\times10^{-7}\text{ s}$$

The total time of transit of electron from cathode to anode = $1\times10^{-9} + 1.816\times10^{-7} = 1.826\times10^{-7}$ s

3.5 FORCE IN A MAGNETIC FIELD

The force acting on a conductor kept in a magnetic field is given by

$$f_m = B\,I\,L \tag{3.19}$$

where f_m = force acting on the conductor, N
B = magnetic flux density, Wb/m^2
I = current in the conductor, A
L = length of the conductor, m

FLASH CARD

What You Learn Here
Examine the force acting on a conductor kept in a magnetic field.

The direction of force is perpendicular to the plane consisting of the components of B and I that are mutually perpendicular, and is directed along the advance of a right-handed screw as illustrated in Fig. 3.3.

If an electron takes T seconds to travel a distance of L metre in the conductor, the total number of electrons passing through any cross-section of conductor in unit time is N/T, where N is the total number of electrons contained in the conductor. Thus the total charge per second crossing any point, i.e., the current is

$$I = \frac{Nq}{T} \tag{3.20}$$

$$f_m = BIL = \frac{BNqL}{T} \tag{3.21}$$

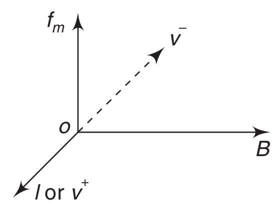

Fig. 3.3 *Illustration of right-handed screw role*

The factor L/T is the drift speed v m/s of the electrons. Hence, the force per electron is

$$f_m = qBv \tag{3.22}$$

If the particle is a positive ion, the direction of current and drift velocity are the same. If the particle is electron, the direction of the current is opposite to that of the drift velocity.

3.6 MOTION IN A MAGNETIC FIELD

The magnetic force acting on a charged particle in a uniform magnetic field can be expressed as

$$\bar{f}_m = q\,\bar{B}\times\bar{v} \tag{3.23}$$

$$f_m = qBv\sin\theta \tag{3.24}$$

FLASH CARD

What You Learn Here
Describe the motion of an electron in a magnetic field.

where φ is the angle between the direction of the magnetic field and the direction of motion of the particle. From Eq. (3.24), we see that the magnitude of the magnetic force is proportional to the charge of the particle, the magnetic flux density, the speed of the charged particle and the angle between the directions of motion of the charged particle and the magnetic flux density.

If an electron is placed in a uniform magnetic field with zero initial velocity, then the magnetic force on the electron is zero, in accordance with Eq. (3.24). If the electron moves along the direction of magnetic flux density, then the angle is zero and the magnetic force is zero. A particle whose initial velocity has no component normal to a uniform magnetic field will continue to move with constant velocity along the lines of flux since the magnetic force on the particle is zero.

The magnetic force acting on an electron moving perpendicularly to the direction of the magnetic flux density is illustrated in Fig. 3.4. The magnetic field is perpendicular to the plane of paper and directed towards the reader. The electron is shown to enter the magnetic field from no-field region with an initial velocity v_o. The direction of the current is opposite to that of the motion. Hence, the magnetic force is upwards and the electron will have a change in its direction of motion. At every point in the magnetic field, a force acts on the electron and the resultant direction will be perpendicular to both the magnetic field and the direction of motion of electron at that point. At every instant, the force f_m is perpendicular to the direction of the particle and no work is done on the electron. This means that the kinetic energy is unaltered and the speed remains the same.

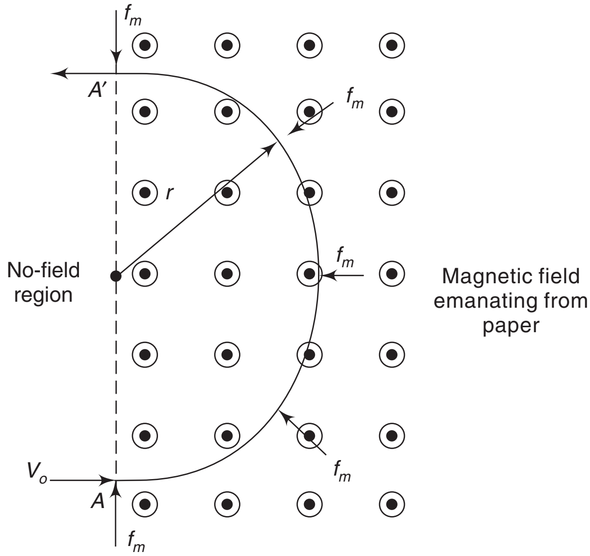

Fig. 3.4 *Motion of an electron in a magnetic field*

This type of force makes an electron move in a circular path with uniform speed. As shown in the figure, the direction of the magnetic force is always towards the centre, O, of the circle. This force is same as the centripetal force which always tries to push the electron towards the centre. Then

$$\frac{mv^2}{r} = qBv \tag{3.25}$$

where 'r' is the radius of circular path of the electron.

From Eq. (3.25),

$$r = \frac{mv^2}{qBv} = \frac{mv}{qB} \tag{3.26}$$

The angular velocity of the electron in radians per second is given by

$$\omega = \frac{v}{r} = \frac{qB}{m} \tag{3.27}$$

The time for one revolution is

$$T = \frac{2\pi}{\omega} = \frac{2\pi m}{qB} \text{ seconds} \tag{3.28}$$

The time taken by the particle to complete one revolution is also called the period.

We see from Eq. (3.26) that the radius is directly proportional to the velocity of the particle. The particles that move faster will traverse in larger circles and if the velocity is less the particles will be bent sharply in smaller circular paths.

Whenever a charged particle enters a uniform magnetic field, with an initial velocity, v, not perpendicular to the magnetic field but makes an angle with the direction of field, as shown in Fig. 3.5, then the velocity can be resolved into two components, $v \sin \theta$, the velocity component perpendicular to the field and $v \cos \theta$, parallel to the field. These two velocity components make the particle to move simultaneously in two directions.

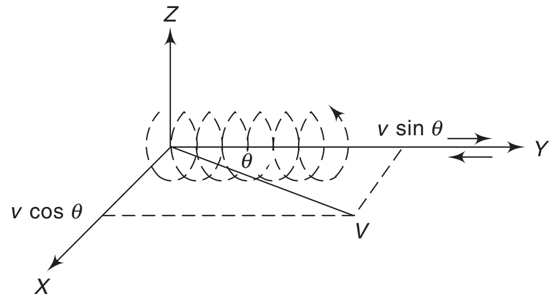

Fig. 3.5 *Motion of an electron emerging in field at an angle θ*

The velocity component parallel to the direction of flux makes the particle move linearly along the direction of flux with no change in velocity. The perpendicular component of velocity makes the particle to move in a circular path as discussed above, with no change in the speed. The net effect, due to these concurrent motion, is that the electron takes a helical path as shown.

The pitch of the helix is defined as the distance travelled by an electron along the direction of the magnetic field in one revolution, and is given by

$$p = v_{oy}\, T \tag{3.29}$$

where T is the time for one revolution and v_{oy} is the initial velocity in the Y-direction. From Eq. (3.28),

$$p = \frac{2\pi m}{qB} v_{oy} \tag{3.30}$$

EXAMPLE 3.8

An electron is accelerated through a potential of 40 V before it enters a magnetic field density of 0.91 Wb/m^2 at an angle of 30 degrees with the field. Find the position of the electron after which it has completed one revolution in the field.

Solution The velocity of the electron is $= \sqrt{\dfrac{2qV_a}{m}} = 3.68 \times 10^6$ m/s

The time taken for one revolution is $T = \dfrac{2\pi m}{Bq} = 4 \times 10^{-11}$ s

The pitch $= T\, v \cos \theta = 1.27 \times 10^{-4}$ m

Thus, the electron has travelled 1.27×10^{-4} m.

EXAMPLE 3.9

A charged particle having charge thrice that of an electron and mass twice that of an electron is accelerated through a potential difference of 50 V before it enters a uniform magnetic field of flux density of 0.02 Wb/m^2 at an angle of 25° with the field. Find (a) the velocity of the charged particle before entering the field, (b) radius of the helical path, and (c) time of one revolution.

Solution The charge of the particle is, $Q = 3\ q$ and the mass of the particle is, $M = 2$ m.

(a) The velocity of the charged particle before entering the field is,

$$v = \sqrt{\frac{2aV}{M}}\sqrt{\frac{2(3q)V}{(2m)}} = \sqrt{\frac{6qV}{2m}}$$

$$= \sqrt{\frac{6 \times 1.602 \times 10^{-19} \times 50}{2 \times 9.1 \times 10^{-31}}}$$

$$= 5.14 \times 10^{6} \text{ m/s.}$$

(b) The radius of the helical path is,

$$r = \frac{Mv \sin\theta}{QB} = \frac{2\ mv \sin\theta}{3qB}$$

$$= \frac{2 \times 9.1 \times 10^{-31} \times 5.14 \times 10^{6} \times \sin 25°}{0.02 \times 3 \times 1.602 \times 10^{-19}}$$

$$= 0.411 \text{ mm.}$$

(c) Time for one revolution,

$$T = \frac{2\pi M}{BQ} = \frac{2\pi(2m)}{B(3q)}$$

$$= \frac{4\pi \times 9.1 \times 10^{-31}}{0.02 \times 3 \times 1.602 \times 10^{-19}}$$

$$= 1.19 \times 10^{-9} \text{ s}$$

EXAMPLE 3.10

An electron is placed in a magnetic field with a period of rotation $T = \frac{35.5}{B} \times 10^{-12}$ s, so that the trajectory of an electron is a circle.

(a) What is the radius described by an electron placed in a magnetic field, perpendicular to its motion, when the accelerating potential is 900 V, and $B = 0.01$ Wb/m^2? (b) What is the time period of rotation?

Solution Given $T = \frac{35.5}{B} \times 10^{-12} s$, $B = 0.01$ Wb/m^2, $V_a = 900$ V

Therefore, $T = 3.55 \times 10^{-9}$ s

Velocity, $$v = \sqrt{\frac{2qV_a}{m}} = \sqrt{2 \times 1.76 \times 10^{11} \times 900} = 17.799 \times 10^{6} \text{ m/s}$$

Radius, $$r = \frac{mv}{qB} = \frac{v}{(q/m)B} = \frac{17.799 \times 10^{6}}{1.76 \times 10^{11} \times 0.01} = 10.11 \text{ mm}$$

3.7 PARALLEL ELECTRIC AND MAGNETIC FIELDS

When both electric and magnetic fields act simultaneously on an electron, no force is exerted due to the magnetic field and the motion of the electron is only due to the electric field intensity E. The electron moves in a direction parallel to the fields with a constant acceleration. If the electric field is along the Y axis and magnetic field is along the $-Y$ axis, the motion of the electron is specified by

FLASH CARD

What You Learn Here
Determine the effect of parallel electric and magnetic fields on the motion of an electron.

$$v_y = v_{oy} - at$$

Integrating Eq. (3.31), we get

$$y = v_{oy}t - \frac{1}{2}at^2 \tag{3.31}$$

where $a = \dfrac{qE}{m}$ is the magnitude of the acceleration.

If a component of velocity v_{ox} perpendicular to the magnetic field exists, initially, this component along with the magnetic field will set the electron in a circular motion. The radius of the circular path is independent of the electric field, but the velocity along the field changes with time. As a result of this, the electron travels in a helical path with the pitch changing with time.

3.8 PERPENDICULAR ELECTRIC AND MAGNETIC FIELDS

Consider an electron starting from rest at the origin. Let the magnetic field be directed along the $-Y$-direction and the electric field be directed along the $-X$-direction. The electron is directed along the $-X$-axis due to the electric field. The force due to the magnetic field is always normal to B, and there is no component of force along the Y-direction, and the Y-component of acceleration is zero. Thus, the motion along Y is given by

FLASH CARD

What You Learn Here
Determine the path of an electron in perpendicular electric and magnetic fields.

$$f_y = 0 \qquad v_y = v_{oy} \qquad y = v_{oy}\, t \tag{3.32}$$

assuming that the electron starts at the origin.

Since the initial velocity is zero, the initial magnetic force is zero and due to the electric field, the electron is directed along the $+X$-axis. As the electron is accelerated in the $+X$-direction, the force due to the magnetic field is no longer zero. There will be a component of this force which is proportional to the X-component of velocity and will be directed along the $+Z$-axis. The path will thus bend away from the $+X$-direction towards the $+Z$-direction. The electric and magnetic forces interact with one another and the net force will finally make the electron to travel in a cycloidal path.

To Determine the Path of the Electron

The force due to electric field $\overline{E}$ is eE along the $+X$-direction. The force due to magnetic field $\overline{B}$ is eBv_x in the $+Z$-direction and eBv_z in the $-X$-direction. Since $\overline{B}$ is in the Y-direction, there is no force on the electron due to v_y. From Newton's law, the force in terms of three components is expressed by

$$f_x = m\frac{dv_x}{dt} = eE - eBvz$$

$$f_y = 0$$

$$f_z = m\frac{dv_z}{dt} = eBv_x$$

Letting $\omega = \dfrac{eB}{m}$ and $u = \dfrac{E}{B}$, we get

$$\frac{dv_x}{dt} = \omega u - \omega v_z \tag{3.33}$$

$$\frac{dv_z}{dt} = +\omega v_x \tag{3.34}$$

Differentiating Eq. (3.33) and combining with Eq. (3.34), we get

$$\frac{d^2 v_x}{dt^2} = -\omega\frac{dc_z}{dt} = -\omega^2 v_x$$

Solving this linear differential equation, with the initial conditions $v_x = v_z = 0$, we get

$$v_x = u \sin \omega t$$

$$v_z = u - u \cos \omega t$$

Integrating each of the above equations, with initial conditions $x = z = 0$, we get the coordinates x and z as

$$x = \frac{u}{w}(1 - \cos \omega t)$$

$$z = ut - \frac{u}{\omega}\sin \omega t$$

Letting $\theta = \omega t$ and $Q = \dfrac{u}{\omega}$, then

$$x = Q(1 - \cos \theta) \tag{3.35}$$

$$z = Q(1 - \sin \theta) \tag{3.36}$$

- ***Cycloidal Path*** The parametric equations (3.35) and (3.36) of a common cycloid are defined as the path generated by a point on the circumference of a circle of radius Q which rolls along a straight line, the Z-axis. This is shown in Fig. 3.6.

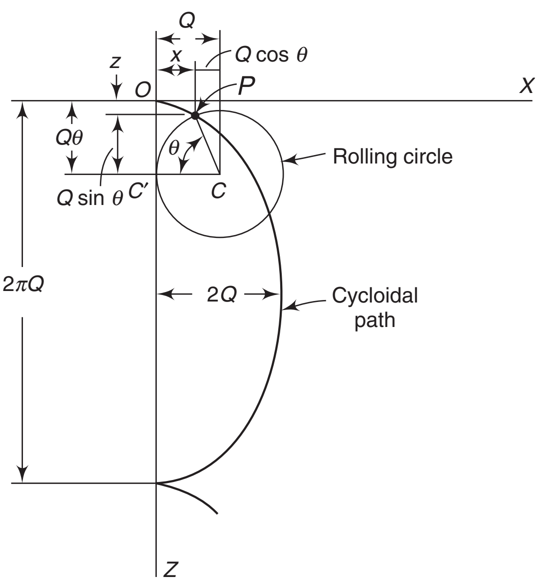

Fig. 3.6 *Cycloidal path of an electron in perpendicular electric and magnetic fields at zero initial velocity*

In Fig. 3.6, P is the position of the electron at any time along the coordinates x and z ($y = 0$), Q is the radius of the rolling circle, θ is the number of rotations through which the circle has rotated and ω is the angular velocity of rotation of the rolling circle.

From Fig. 3.6, it can be written as

$$x = Q - Q \cos \theta$$

$$z = Q\theta - Q \sin \theta$$

where these equations are similar to equations (3.35) and (3.36). As illustrated in Fig. 3.6, $2Q$ is the maximum distance that the electron covers along X-axis, and it is equal to the diameter of the rolling circle. $2\pi Q$ is the distance covered along Z-axis, and it is equal to the circumference of the rolling circle.

- ***Cycloidal Helical Motion*** The particle will have a constant velocity normal to plane through the projection of the path remains cycloid, when an initial velocity exists and it is directed parallel to the magnetic field. This path is called *cycloidal helical motion*.

- ***Straight-line Path*** When the electron is directed perpendicular to both the magnetic and electric fields, and if the total force on electron is zero, it moves only along the Z-axis. Therefore,

$$v_{ox} = 0, v_{oy} = 0, v_{oz} \neq 0$$

$$f_x = 0$$

$$eE - eBv_{oz} = 0$$

Therefore, $$v_{oz} = \frac{E}{B}u$$

The velocity u does not depend on the charge or mass of the ions. The perpendicular electric and magnetic fields act as some sort of velocity filter, and only the particles having the velocity ratio of E/B get selected.

EXAMPLE 3.11

The magnetic flux density $B = 0.02$ Wb/m^2 and electric field strength $E = 10^5$ V/m are uniform fields, perpendicular to each other. A pure source of an electron is placed in a field. Determine the minimum distance from the source at which an electron with 0 V will again have 0 V in its trajectory under the influence of combined electric and magnetic fields.

Solution Given, $B = 0.02$ Wb/m^2, $E = 10^5$ V/m

The initial velocity, $v_o = 0$

Referring to Fig. 3.6, the minimum distance from the source at which an electron with 0 V will again have 0 V in its trajectory under the influence of combined electric and magnetic fields is given by

$$2\pi Q = \frac{2\pi u}{\omega}$$

where $Q = \frac{u}{\omega}$

ω = angular velocity of rotation of the rolling circle
θ = number of radians through which the circle has rotated
Q = radius of the rolling circle
u = velocity of translation of the center of the rolling circle

Here, $$\omega = \frac{eB}{m} \text{ and } u = \frac{E}{B}$$

We know that, $e = 1.602 \times 10^{-19}$ C and $m = 9.109 \times 10^{-31}$ kg $= 9.109 \times 10^{-28}$ g

Therefore, $$\omega = \frac{eB}{m} = \frac{1.602 \times 10^{-19} \times 0.02}{9.109 \times 10^{-28}} = 3.5174 \times 10^9 \text{ rad/sec}$$

and $$u = \frac{E}{B} = \frac{10^5}{0.02} = 5 \times 10^6 \text{ m/sec}$$

Hence, the minimum distance travelled by an electron in perpendicular electrical magnetic fields, when the, initial velocity is 0, is given by

$$\frac{2\pi u}{\omega} = \frac{2\pi \times 5 \times 10^6}{3.5174 \times 10^9} = 8.932 \times 10^{-3} \text{ m}$$

3.9 ELECTROSTATIC DEFLECTION IN A CATHODE-RAY TUBE

FLASH CARD

What You Learn Here
Describe the electrostatic deflection of an electron beam in a cathode-ray tube.

The electrostatic deflection system uses a pair of deflection plates as shown in Fig. 3.7. The deflecting voltages are applied between the two plates. For deflecting the beam in the horizontal and vertical directions, two sets of plates are required. The horizontal deflection plates are kept vertically and the vertical deflection plates are kept horizontally, however, the plane of the plates are kept parallel to the axis of the cathode-ray tube. The electrons are attracted towards the positive plates and when they leave the region below the plates, they travel in straight line, at an angle with the axis. Then the electron beam strikes the screen at a point.

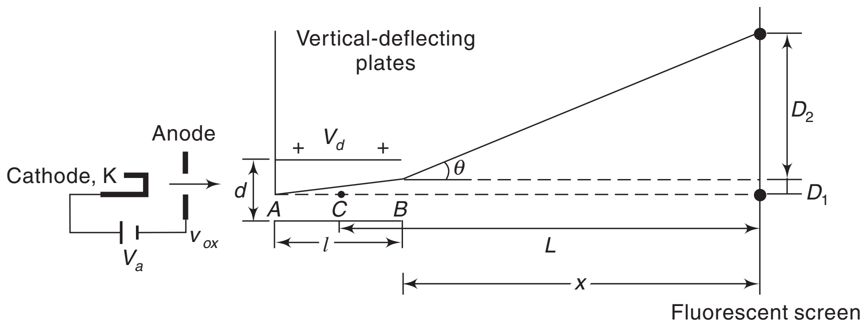

Fig. 3.7 *Electrostatic deflection system in a CRT*

Electrostatic deflection sensitivity of a pair of deflection plates of a CRT is defined as the amount of deflection (in mm or inch) of electron spot produced when a voltage of 1 V dc is applied between the corresponding deflection plates. The deflection sensitivity may be different for *X-X* and *Y-Y* plates. The amount of deflection produced may be calculated knowing (i) the dimensions of the tube components, and (ii) deflecting voltage.

Figure 3.7 shows the configuration of the electrostatic deflection system in a CRT. Two plates of length l and spacing d are kept at a distance x from the screen. Let the voltage applied between the plates be V_d volt, v_{ox} be the velocity of an electron on entering the field of deflection plates.

Then, $$v_{ox} = \sqrt{\frac{2q}{m} \cdot V_a} = 5.94 \times 10^5 \sqrt{V_a} \text{ m/s}$$

where V_a is the final anode voltage (in volt), q is the charge of an electron in coulomb, and m is the mass of an electron in kilogram.

As the beam passes through the field of the deflection plates, the electrons are attracted towards the positive plate by a force equal to

$$F = \frac{V_d \cdot q}{d}$$

The force produces an acceleration of 'a' m/s^2 and is equal to the force divided by mass m of an electron. This mass may be supposed to be the mass at rest because the deflecting voltage is hardly even greater than 2000 V.

$$a = \frac{V_d \cdot q}{dm}$$

The forward motion however continues at the velocity v_{ox}. The time taken by an electron to traverse the field of the plates is then $\dfrac{l}{v_{ox}}$ s. The upward velocity attained by an electron in this time interval $\dfrac{l}{v_{ox}}$ is v_y. Then,

$$v_y = \frac{l}{v_{ox}} \cdot a = \frac{l}{v_{ox}} \cdot \frac{V_d \cdot q}{dm}$$

Then the ratio of upward velocity to forward velocity at the instant the electron leaves the field is

$$\frac{v_y}{v_{ox}} = \frac{V_d \cdot q \cdot l}{dm\, v_{ox}^2}$$

The electron follows a curved path from A, the point of entrance, to the point B, the point leaving the field. Let the vertical displacement during this be D_1. Then D_1 is given by

$$D_1 = \frac{1}{2} a \left(\frac{l}{v_{ox}} \right)^2 = \frac{1}{2} \cdot \frac{V_d \cdot q \cdot l^2}{dm\, v_{ox}^2}$$

If θ is the angle with the axis that the electron beam makes after emerging out from the field of the deflection plates, then

$$\tan \theta = \frac{v_y}{v_{ox}} = \frac{D_2}{x}$$

(or) $$D_2 = L \cdot \frac{v_y}{v_{ox}} = \frac{V_d}{d} \cdot \frac{q}{m} \cdot \frac{1}{v_{ox}^2} \cdot x$$

Total deflection $D = D_1 + D_2 = \dfrac{V_d \cdot ql[l/2 + x]}{dm\, v_{ox}^2} = \dfrac{V_d \cdot qlL}{dm\, v_{ox}^2}$

The total deflection of the spot will be same as an electron travelling a straight-line path at an angle θ from the point C instead of B. Since $L = \dfrac{1}{2} + x$, $D = \dfrac{V_d\, ql\, L}{dm\, v_{ox}^2}$.

But, $$v_{ox}^2 = \frac{2qV_a}{m}$$

where V_a is the final anode voltage.

$$D = \frac{V_d}{V_a} \times \frac{lL}{2d}$$

For a given CRT, the quantities *l, L*, and *d* are fixed. Hence, the deflection, *D* of the spot can be changed only by changing the ratio $\frac{V_d}{V_a}$. The electrostatic deflection sensitivity of a CRT is defined as the deflection in metre on the screen per volt of deflecting voltage.

$$\text{Deflection sensitivity,} \quad S = \frac{D}{V_d} = \frac{lL}{2dV_a}, \text{ cm/V} \tag{3.37}$$

where l = length of the deflecting plates, m
L = distance from the centre of the plate to the screen, m
V_a = accelerating potential, V
d = separation between the two plates, m

The deflection sensitivity is (i) directly proportional to the length of the deflection plates *l*, (ii) directly proportional to the distance *L* between the screen and the centre of the deflection plates, and (iii) inversely proportional to the spacing *d* between the deflection plates and inversely proportional to the final anode voltage V_a.

EXAMPLE 3.12

An electrostatic cathode ray tube has a final anode voltage of 600 V. The deflection plates are 3.5 cm long and 0.8 cm apart. The screen is at a distance of 20 cm from the centre of plates. A voltage of 20 V is applied to the deflection plates. Calculate (a) velocity of electron on reaching the field, (b) acceleration due to deflection field, (c) deflection produced on the screen in cm, and (d) deflection sensitivity in cm/V.

Solution Given $V_a = 600$ V, $l = 3.5$ cm, $d = 0.8$ cm, $L = 20$ cm, $V_d = 20$ V

(a) The velocity of the electron, $v = \sqrt{\frac{2qV_a}{m}} = 14.5 \times 10^6 \text{ ms}^{-1}$

(b) $ma = qE$
Thus, acceleration, $a = qE/m = (q/m)(V_d/d) = 43.95 \times 10^{13} \text{ ms}^{-2}$

(c) The deflection on the screen, $D = ILV_d/2V_ad = 1.45$ cm

(d) Deflection sensitivity = $D/V_d = 0.0725$ cm/V.

EXAMPLE 3.13

In a CRT, the deflection plates are 2 cm long and are spaced 0.5 cm apart. The screen is 20 cm away from the centre of the deflecting plates. The final anode voltage is 800 V. Calculate (a) the velocity of the beam on emerging from the field, and (b) the voltage that must be applied to the deflecting plates to have a displacement of 1 cm.

Solution Given $V_a = 800$ V, $l = 2$ cm, $d = 0.5$ cm, $L = 20$ cm, $D = 1$ cm

(a) The velocity of the beam, $v = \sqrt{\frac{2qV_a}{m}} = \sqrt{\frac{2 \times 1.602 \times 10^{-19} \times 800}{9.1 \times 10^{-31}}} = 16.8 \times 10^6$ m/s

(b) The deflection of the beam, $D = \frac{lLV_d}{2dV_a}$

i.e., $$1 \times 10^{-2} = \frac{2 \times 10^{-2} \times 20 \times 10^{-2} \times V_d}{2 \times 0.5 \times 10^{-2} \times 800}$$

Therefore, the voltage that must be applied to the plates,

$$V_d = 20 \text{ V}$$

EXAMPLE 3.14

In an electrostatic deflecting CRT, the length of the deflection plate is 2 cm and the spacing between deflecting plates is 0.5 cm. The distance from the centre of the deflecting plate to the screen is 20 cm and the deflecting voltage is 25 V. Find the deflection sensitivity, the angle of deflection and velocity of the beam. Assume final anode potential is 1000 V.

Solution Given, in an electrostatic deflecting CRT,

Length of deflection plate, $l = 2$ cm

Spacing between deflecting plates, $d = 0.5$ cm

Distance from center of deflecting plate to screen, $L = 20$ cm

Deflecting potential, $V_d = 25$ V

Anode potential, $V_a = 1000$ V

(a) Velocity of beam, $$v = \sqrt{\frac{2qV_a}{m}} = \sqrt{\frac{2 \times 1.602 \times 10^{-19} \times 1000}{9.1 \times 10^{-31}}} = 18.75 \times 10^6 \text{ ms}^{-1}$$

(b) Deflection sensitivity $= \dfrac{D}{V_d}$

where $$D = \frac{ILV_d}{2V_a d} = \frac{2 \times 10^2 \times 20 \times 10^{-2} \times 25}{2 \times 1000 \times 0.5 \times 10^{-2}} = 10^{-2} \text{ cm}$$

Therefore, the deflection sensitivity $= \dfrac{10^{-2}}{25} = 0.0004$ cm/V

(c) To find the angle of deflection, θ:

$$\tan\theta = \frac{D}{(L-1)} = \frac{10^{-2}}{20-2} = \frac{10^{-2}}{18}$$

Therefore, $$\theta = \tan^{-1}\left(\frac{1}{1800}\right) = 0.0318°$$

EXAMPLE 3.15

An electron with a velocity of 3×10^5 ms^{-1} enters an electric field of 910 V/m making an angle of 60° with the positive direction. The direction of the electric field is in the positive y-direction. Calculate the time required to reach its maximum height.

Solution Given $v_0 = 3 \times 10^5$ m/s

$E = 910$ V/m

$\theta = 60°$

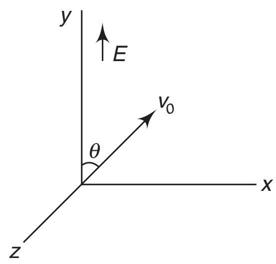

Fig. 3.8

The electron starts moving in the +y-direction, but, since acceleration is along the –y-direction, its velocity is reduced to zero at time $t = t'$.

$$v_{0y} = v_0 \cos\theta = 3 \times 10^5 \times \cos 60° = 0.15 \times 10^6 \text{ m/s}$$

$$a_y = \frac{qE}{m} = \frac{1.602 \times 10^{-19}}{9.109 \times 10^{31}} \times 910 = 1.5984 \times 10^{14} \text{ m/s}^2$$

$$t' = \frac{v_{0y}}{a_y} = \frac{0.15 \times 10^6}{1.5984 \times 10^{14}} = 9.384 \times 10^{-10} \text{ s} = 0.938 \text{ ns}$$

3.10 MAGNETIC DEFLECTION IN A CATHODE-RAY TUBE

FLASH CARD

What You Learn Here
Describe the magnetic deflection of an electron beam in a CRT.

The applied magnetic field is perpendicular to the direction of the electron beam. The force exerted on the electron beam by the magnetic field bends the electron beam in a direction perpendicular to both the field and the direction of electron movement.

Figure 3.9 shows the magnetic deflection system in a CRT. In the region of the uniform magnetic field, the electron experiences a force qBv, where v is the speed. The path OM is the arc of a circle whose center is at Q. The velocity of the charged particles before entering into the field remains constant, which is given by

$$v = v_{cx} = \sqrt{\frac{2qV_a}{m}} \tag{3.38}$$

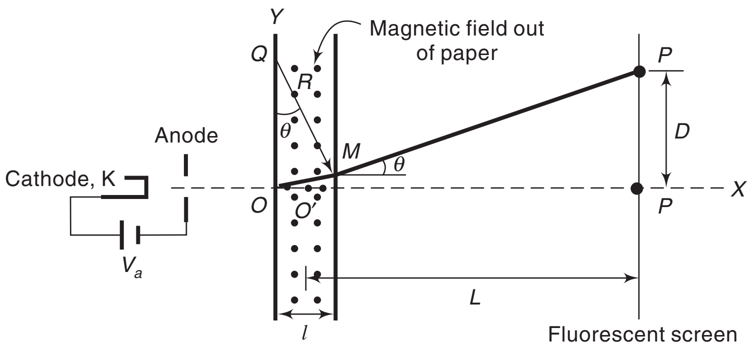

Fig. 3.9 *Magnetic deflection system in a CRT*

The small angle of deflection θ is equal to the length of the arc OM divided by R, the radius of the circle. Then,

$$\theta \approx \frac{1}{R} \tag{3.39}$$

The magnetic field continues to deflect the electron beam at right angles to its movement and hence the path taken by the electron beam within the magnetic field is a part of a circle whose radius, R, is given by

$$R = \frac{mv}{qB} \tag{3.40}$$

where B = magnetic flux density, Wb/m^2
v = velocity of the electron beam, m/s
m = the mass of an electron, kg, and
q = the charge of an electron, C

Usually, L is far greater than l, so that small error will be made in assuming that the straight line MP', if projected backward, will pass through the center O' of the region of the magnetic field. Then

$$D \approx L \tan\theta \approx L\theta \tag{3.41}$$

Using equations (3.38) to (3.40), the above equation becomes

$$D \approx L\theta = \frac{lL}{R} = \frac{lLqB}{mv} = \frac{lLB}{\sqrt{V_a}}\sqrt{\frac{q}{2m}}$$

The magnetic deflection sensitivity of a CRT is defined as the ratio of the deflection to the unit magnetic field intensity. The magnetic deflection sensitivity is given by

$$\frac{D}{B} = \frac{lL}{\sqrt{V_a}}\sqrt{\frac{q}{2m}} \text{ mm/Wb/m}^2 \tag{3.42}$$

The magnetostatic deflection sensitivity is also defined as the amount of deflection of the spot caused by a current of 1 mA through the deflection of coil. Thus, magnetostatic deflection sensitivity can also be expressed in cm/mA.

The magnetostatic deflection sensitivity is independent of B. The electrostatic deflection sensitivity varies inversely with the anode voltage, whereas the magnetic deflection sensitivity varies inversely with the square root of the anode voltage. The sensitivity increases with L and hence, for maximum sensitivity, the deflecting coils are placed as far down the neck of the cathode ray tube as possible.

EXAMPLE 3.16

In a CRT, the distance of the screen from the centre of the magnetic field is 20 cm. The deflecting magnetic field of 1×10^{-4} Wb/m^2 flux density extends for a length of 2 cm along the tube axis. The final anode voltage is 800 V. Calculate the deflection of the spot.

Solution The deflection of the spot,

$$D = \frac{lBL}{\sqrt{V_a}}\sqrt{\frac{q}{2m}} = \frac{2\times10^{-2}\times1\times10^{-4}\times20\times10^{-2}}{\sqrt{800}}\sqrt{\frac{1.602\times10^{-19}}{2\times9.1\times10^{31}}} = 0.42 \text{ cm}$$

EXAMPLE 3.17

The electron beam in a CRT is displaced vertically by a magnetic field of 2×10^{-4} Wb/m^2 flux density. The length of the magnetic field along the tube axis is the same as that of the electrostatic deflection plates. The final anode voltage is 800 V. Calculate the voltage which should be applied to the Y-deflection plates 1 cm apart, to return the spot back to the centre of the screen.

Solution The magnetostatic deflection, $D = \frac{lBL}{\sqrt{V_a}}\sqrt{\frac{q}{2m}}$

The electrostatic deflection, $D = \frac{lLV_d}{2\,dV_a}$

For returning the beam back to the centre, the electrostatic deflection and the magnetostatic deflection must be equal, i.e.,

$$\frac{lLV_d}{2dV_a} = \frac{lBL}{\sqrt{V_a}}\sqrt{\frac{q}{2m}}$$

Therefore, $$V_d = dB\sqrt{\frac{2V_a q}{m}} = 1\times10^{-2}\times 2\times10^{-4}\times\sqrt{\frac{2\times800\times1.602\times10^{-19}}{9.1\times10^{-31}}} = 33.56 \text{ V.}$$

3.11 MAGNETIC FOCUSING

FLASH CARD

What You Learn Here
Describe the concept of magnetic focusing for accurate focusing of an electron beam in a magnetic field.

Consider a cathode-ray tube placed in a constant longitudinal magnetic field such that the axis of the tube coincides with the direction of the magnetic field. A magnetic field is obtained through the use of a long solenoid placed within the coil. Figure 3.10 shows the helical path of an electron in magnetic field in which the axis of the cathode-ray tube is along the *Y*-axis, origin '*O*' is the point at which the electrons emerge from the anode, v_o is the velocity at the origin, v_{ox} is the initial transverse velocity due to the mutual repulsion of the electrons. The resultant path is a helix whose axis is parallel to the *Y*-axis and displaced by a distance '*r*' along the *Z*-axis, as shown in Fig. 3.10.

The pitch of the helix, defined as the distance traveled along the direction of the magnetic field in one revolution, is given by

$$p = \frac{2\pi m}{qB} v_{oy}$$

When the applied magnetic field is zero, the electron beam is defocused and a smudge is seen on the screen. This results in the formation of a broad, faintly illuminated area instead of a bright point on the screen. As the magnetic field is increased from zero, the electrons take helical paths of different radius since the velocity v_{ox} that controls the radius of the path will be different for different electrons. But the time to trace the electron path is independent of v_{ox}, and so the period will be the same for all electrons. If the distance from the anode to the screen is made equal to one pitch, all the electrons will be brought back to the *Y*-axis (the point *O′* in Fig. 3.10),

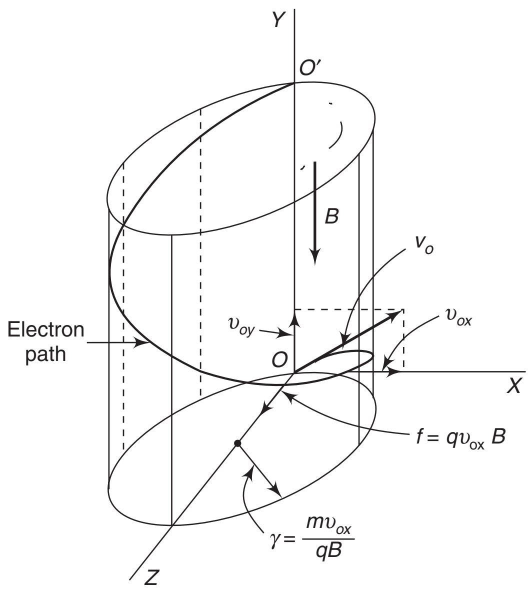

Fig. 3.10 *Helical path of an electron with a constant magnetic field*

since they all will have made just one revolution. Under these conditions, an image of the anode hole will be observed on the screen.

As the field is increased from zero, the smudge on the screen will contract and will become a tiny sharp spot when a critical value of the field is reached. This critical field is that which makes the pitch of the helical path just equal to the anode-screen distance. The pitch of the helix decreases by increasing the strength of the field beyond this critical value and the electrons travel through more than one complete revolution. The electrons then strike the screen at various points, so that a defocused spot is again visible. A magnetic field strength will ultimately be reached at which the electrons make two complete revolutions in their path from the anode to the screen, and once again the spot will be focused on the screen.

If the screen is perpendicular to the *Y*-axis at a distance '*L*' from the point of emergence of the electron beam from the anode, then, the electron beam will come to a focus at the center of the screen provided that *L* is an integral multiple of *p*.

By substituting $v_{oy} = \sqrt{\frac{2qV_a}{m}}$ and $L = np$ in the previous equation and simplifying, we get

$$\frac{q}{m} = \frac{8\pi^2 V_a n^2}{L^2 B^2}$$

where n is an integer representing the order of the focus, B is the magnetic field, and V_a is the anode cathode potential. This arrangement is used to measure the ratio q/m for electrons very accurately. But, this method of employing a longitudinal magnetic field over the entire length of a commercial tube is not too practical.

For accurate focusing of the electron beam, a short coil is wound around the neck of the tube in a commercial tube. A radial component of B exists in addition to the component along the tube axis due to the fringing of the magnetic lines of flux. Hence, there are two components of force on the electron, one due to the axial component of velocity and the radial component of the field, and the other due to the radial component of the velocity and the axial component of the field. It can be understood that the motion will be rotation about the axis of the tube and the electron on leaving the region of the coil may be turned sufficiently so as to move in a line toward the center of the screen. A rough adjustment of the focus is obtained by positioning the coil properly along the neck of the tube. The fine adjustment of focus is done by controlling coil current and the electron can be focused accurately towards the centre of the screen by this method.

3.12 COMPARISON BETWEEN ELECTRIC AND MAGNETIC DEFLECTION SYSTEMS

The electrostatic and magnetostatic deflection systems possess different properties and each has its own merits and demerits. Magnetic deflection uses coils resulting in losses and also requires large currents for deflections. On the other hand, the electrostatic deflection uses plates and require very little power. Generally, the magnetic flux density is directly proportional to the current flowing, and deflection produced is a direct function of the current in the coils. The current in a coil is a function of time integral of the applied voltage and because of this reason, the magnetic deflection cannot be used for direct visual display of the applied voltage.

> **FLASH CARD**
>
> **What You Learn Here**
> Distinguish between electric and magnetic deflections and describe the defects in deflection.

The electrostatic deflection can be used at much higher frequencies and, therefore, preferred for high-frequency instruments. The electrostatic deflection sensitivity decreases more rapidly with increased anode voltage than in the case of the magnetic deflection. Therefore, the accelerating potential can be increased to give brighter spot with small decrease in sensitivity for magnetic deflection than for electric deflection. In electrostatic deflection, the degree of deflection is proportional to square of electron velocity. In contrast, in tubes using magnetostatic deflection system, the deflection is inversely proportional to electron velocity. Hence, the electrostatic deflection suffers deflection defocusing when the beam is bent through a large angle. For these reasons, the magnetic deflection is preferred in cathode ray tube used in television because large permissible angle of deflection reduces the length of the tube for a given diameter.

- ***Defects of Deflection*** When an electron beam is deflected from the axial direction, the spot on the fluorescent screen tends to distort and enlarge. This phenomenon is commonly referred to as *deflection defocusing*. This may be due to (i) non-uniform distance between the electron gun and the different parts of the screen, (ii) non-uniform electric and magnetic deflection fields, and (iii) unequal velocities of electrons in the beam. With non-uniform deflecting fields, the part of the beam which passes through region of weaker field is less deflected than the part of the beam which passes through region of stronger field. If the electrons are emitted from cathode with unequal initial velocities, as the degree of deflection is also a function of the electron velocity, electrons with different velocities are deflected differently which forms a blurred elliptical spot on the screen.

PROBE

Analyze the reason why deflection defocusing occurs.

Key Terms

Deflection defocusing
Electrostatic deflection
Electrostatic deflection sensitivity of a CRT
Field intensity
Force
Magnetostatic deflection sensitivity
Pitch of helix
Potential energy
Sensitivity of a pair of deflection plates

Review Questions

1. Define the terms: (i) Electric field intensity (ii) Electric potential (iii) Potential energy.
2. State the law of conservation of energy.
3. What are the assumptions made while analyzing the motion of an electron in an electric field?
4. Discuss the motion of an electron between two parallel plates under the influence of applied potential.
5. Two plane parallel plates *A* and *B* are placed 5 mm apart and potential of *B* is made 300 V positive with respect to plate *A*. An electron starts from rest from plate *A*. Calculate (i) kinetic energy of the electron on reaching plate *B*, (ii) the velocity of the electron on reaching plate *B*, and (iii) time taken for the electron to reach plate *B*. [*Ans*. (i) 4.8×10^{-17} joule (ii) 10.28×10^{6} m/s (iii) 0.973×10^{-9} s]
6. Two parallel plates are kept 8 mm apart. One plate is kept 250 V positive with respect to the other. An electron starts from rest from the negative plate. Calculate the velocity, kinetic energy and distance travelled after time 0.4×10^{-19} s. [*Ans*. 2.2×10^{6} m/s; 22.84×10^{-19} joule; 0.44 mm]
7. In a vacuum diode, the spacing between the parallel plane plates of cathode and anode is 5 mm and the potential difference is 250 V. Calculate the time taken by the electron, with an initial velocity of 1×10^{6} m/s, to travel from cathode to anode. [*Ans*. 0.96 ns]
8. Two parallel plates are kept a distance 12 mm apart. One plate is 800 V positive with respect to the other. An electron starts from rest from the negative plate. Find the distance travelled, time taken and the kinetic energy of the electron when it has acquired a speed of 5×10^{6} m/s. At this instant, the potential across the plates is suddenly removed. Find the total time of travel of the electron from the negative plate to the positive plate. [*Ans*. 1.065×10^{-3} m, 0.426×10^{9} s, 113.8×10^{-19} joule, 2.613×10^{-9} s]

9. Discuss the motion of an electron under the influence of applied magnetic field. ○●●

10. An electron moving with initial velocity of 10^6 m/s enters an uniform magnetic field at an angle of 30° with it. Calculate the magnetic flux density required in order that the radius of helical path be 1 m. Also, calculate the time taken by the electron for one revolution and the pitch of the helix. ●●●

[*Ans*. 0.284×10^{-4} Wb/m^2; 12.57×10^{-6} s; 10.88 m.]

11. What happens to an electron when it is exposed to parallel electric and magnetic fields? ○●●

12. Why does an electron take a cycloidal path when it is exposed to perpendicular electric and magnetic fields? ○●●

13. Define cycloidal helical motion. ○○●

14. Define electrostatic deflection sensitivity. ○○●

15. An electrostatic cathode ray tube has a final anode voltage of 400 V. The deflection plates are 2 cm long and 1 cm apart. The screen is at a distance of 10 cm from the centre of the plates. A voltage of 20 V is applied to the deflection plates. Calculate (i) velocity of electron on reaching the field, (ii) acceleration due to deflection field, (iii) deflection produced on the screen, and (iv) deflection sensitivity. ●●●

[*Ans.* (i) 11.9×10^6 ms^{-1} (ii) 35.1×10^{10} ms^{-2} (iii) 0.5 cm (iv) 0.025 cm/V]

16. In a cathode-ray tube having electric deflection system, the deflection plates are 2 cm long and have a uniform spacing of 4 mm between them. The fluorescent screen is 25 cm away from the centre of the deflection plates. Calculate the deflection sensitivity, if the potential of the final anode is (i) 1000 V, (ii) 2000 V, and (iii) 3500 V. [*Ans.* (i) 0.0625 cm/V (ii) 0.03125 cm/V (iii) 0.01785 cm/V] ●●●

17. Define magnetic deflection sensitivity. ○○●

18. In a CRT, the distance of the screen from the centre of the magnetic field is 22 cm. The deflecting magnetic field of flux density 2×10^{-4} Wb/m^2 extends for a length of 2.5 cm along the tube axis. The final anode voltage is 1250 V. Calculate the deflection of the spot in cm. [*Ans*. 0.92 cm] ●●●

19. Compare electrostatic deflection with magnetostatic deflection. ○●●

20. Why is magnetic deflection preferred to electrostatic deflection in the CRT used in television? ●●●

21. Explain in detail about magnetic focusing. ○●●

Note: ○○● Level 1 & Level 2 category
○●● Level 3 & Level 4 category
●●● Level 5 & Level 6 category

Objective-Type Questions

1. The magnitude of an electron is
(a) 6×10^{18} C (b) 1.602×10^{-19} C (c) 1.759×10^{11} C (d) 9.107×10^{-31} C

2. The mass of an electron is
(a) 6×10^{18} kg (b) 1.602×10^{-19} kg (c) 1.759×10^{11} kg (d) 9.107×10^{-31} kg

3. When a charge q is moved in an electric field E, the force on q due to the electric field is
(a) $\frac{q}{E}$ (b) $\frac{E}{q}$ (c) qE (d) none of the above

4. The unit of electric field intensity is
(a) C (b) V/m (c) V/m^2 (d) Wb/m^2

5. If two charged particles are separated by a distance d, then force existing between them is F. What is the relation between F and d?
(a) Linear (b) $1/d$ (c) $1/d^2$ (d) None of the above

6. Force existing between the positive and negative charged particles is
(a) repulsive (b) attractive (c) no force prevent (d) none of the above

7. Force on a unit positive charge is called
(a) force (b) potential (c) field intensity (d) work done

8. Field lines from a positive charge are ________ in nature.
(a) diverging (b) converging (c) no direction (d) none of the above

9. If the particle is a positive ion, then the direction of current and drift velocity is
(a) in the same direction (b) in opposite directions
(c) perpendicular to each other (d) none of the above

10. Magnetostatics is the study of
(a) moving charges (b) static charges
(c) moving charges with uniform speed (d) none of the above

11. The force acting on a conductor of length L, carrying current I, kept in a magnetic field B, is given by
(a) BIL (b) $1/BIL$ (c) BI/L (d) none of the above

12. If B is the magnetic flux density, v is the drift speed of the electron and q is the charge of the electron, then the force per electron is
(a) $\frac{qB}{v}$ (b) qBv (c) $\frac{v}{qB}$ (d) $\frac{q}{Bv}$

13. What is the speed of an electron after it has removed through a potential difference of 1000 V?
(a) 1×10^7 m/s (b) 1.87×10^6 m/s (c) 1.87×10^9 cm/s (d) 3×10^8 m/s

14. In a parallel-plate capacitor containing two plates of areas A_1 and A_2, which area has to be considered in order to calculate capacitance?
(a) Large area out of both (b) Smaller area out of both
(c) Adding of both (d) Common area between two plates

15. The electric field inside a hollow metallic charged sphere is
(a) increasing towards centre (b) decreasing
(c) zero (d) none of the above

16. The unit of electric flux density is
(a) C (b) F/m (c) C/m^2 (d) Wb/m^2

17. Dielectric flux is proportional to
(a) potential difference between electrodes (b) resistivity of medium
(c) rate of change of potential difference (d) rate of change of current

18. If two plates are separated by 10 mm and voltage applied is 1 V, the electric field produced is
(a) 10^8 V/m (b) 10^9 V/m (c) 10^{10} V/m (d) 10^9 V/cm

19. If an emf of 10 mV is induced in a coil of 4 mH inductance, the rate of change of current is
(a) 3.5 A/s (b) 2.5 A/s (c) 5 A/s (d) 10 A/s

20. Electric field intensity at any point in an electric field is equal to the ------ at that point
(a) electric flux (b) magnetic flux density
(c) potential gradient (d) none of the above